Exercise 47

Find the absolute maximum and absolute minimum values of f on the given interval.

$$f(x) = 12 + 4x - x^2, \quad [0, 5]$$

Solution

Take the derivative of the function.

$$f'(x) = \frac{d}{dx}(12 + 4x - x^2)$$

$$= 12(0) + 4(1) - 2x$$

$$= 4 - 2x$$

Set $f'(x) = 0$ and solve for x.

$$4 - 2x = 0$$

$$2x = 4$$

$$x = 2$$

This value of x is within $[0, 5]$, so evaluate f here.

$$f(2) = 12 + 4(2) - (2)^2 = 16 \qquad \text{(absolute maximum)}$$

Now evaluate the function at the endpoints of the interval.

$$f(0) = 12 + 4(0) - (0)^2 = 12$$

$$f(5) = 12 + 4(5) - (5)^2 = 7 \qquad \text{(absolute minimum)}$$

The smallest and largest of these numbers are the absolute minimum and maximum, respectively, over the interval $[0, 5]$.

The graph of the function below illustrates these results.

The graph shows $y = 12 + 4x - x^2$ with a maximum point at $(2, 16)$ and an endpoint at $(5, 7)$.